Vedic Mathematics

by Jeganathan Sriskandarajah†

Sri Bharati Krsna Tirthaji

Vedic Mathematics is based on 16 sutras (or aphorisms) dealing with mathematics related to arithmetic, algebra, and geometry. These methods and ideas can be directly applied to trigonometry, plain and spherical geometry, conics, calculus (both differential and integral), and applied mathematics of various kinds. It was reconstructed from ancient Vedic texts early in the last century by Sri Bharati Krsna Tirthaji. Bharati Krsna was born in 1884 and died in 1960. He was a brilliant Indian scholar with the highest honours in the subjects of Sanskrit, Philosophy, English, Mathematics, History, and Science. When he heard about the parts of the Vedas containing mathematics, he resolved to study these scriptures and find their meaning. Between 1911 and 1918, he was able to decode from the ancient Sutras the mathematical formulae that we now call Vedic Mathematics.

The Sanskrit word Veda means knowledge, and the Vedas are considered the most sacred scripture of Hinduism referred to as sutras, meaning what was heard by or revealed to the seers. Vedas are the most ancient scriptures dealing with all branches of knowledge—spiritual and worldly. Although there is an ongoing dispute regarding the age of the Vedas, it is commonly believed that these scriptures were written at least several centuries BC. The hymns of the Rig Veda are considered the oldest and most important of the Vedas, having been composed between 1500 BC and the time of the great Bharata war, about 900 BC. The Vedas consist of a huge number of documents (there are said to be thousands of such documents in India, many of which have not yet been translated), which are shown to be highly structured, both within themselves and in relation to each other. The most holy hymns and mantras are put together into four collections called the Rig, Sama, Yajur, and Atharva Vedas. They are difficult to date, because they were passed on orally for about 1000 years before they were written down. More recent categories of Vedas include the Brahmanas, or manuals for ritual and prayer. Subjects covered in the Vedas include: grammar, astronomy, architecture, psychology, philosophy, archery, etc.

One hundred years ago, Sanskrit scholars translating the Vedic documents were surprised at the depth and breadth of knowledge contained in them. Some documents, called 'ganita sutras' (the name *'ganita'* means mathematics), were devoted to mathematical knowledge. In these sutras, which, for example, addressed the geometry of construction of sacrificial altars, geometrical figures such as straight lines, rectangles, circles and triangles are discussed in a very profound manner. There are various descriptions of the rules for transformations, including the 'Pythagorean' theorem. The proof of this theorem, as described in the Vedas, is illustrated in Figure 1.

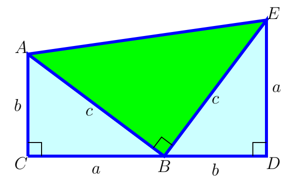

Figure 1

The area of the trapezoid $ACDE$ is equal to the sum of the areas $\triangle ABC + \triangle ABE + \triangle BDE$. Thus,

$$\frac{(a+b)^2}{2} = \frac{ab}{2} + \frac{c^2}{2} + \frac{ab}{2}$$
$$\implies c^2 = a^2 + b^2.$$

The Apollonius theorem, which states *for a triangle with sides a, b, and c and median m to the side with length a, that*

$$b^2 + c^2 = 2m^2 + \frac{a^2}{2},$$

was also described in the Vedas. Its simple proof, as presented in scripture in the Vedas, can be summarized in a few lines (see Figure 2):

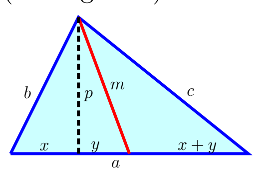

Figure 2

$$\begin{aligned} b^2 + c^2 &= x^2 + p^2 + (x+2y)^2 + p^2 \\ &= 2(y^2+p^2) + 2(x+y)^2 \\ &= 2m^2 + \frac{a^2}{2}. \end{aligned}$$

The areas of a triangle, a parallelogram, and a trapezoid, as well as the volume of a prism, a cylinder, and a pyramid, are also discussed in sutras. The quadratic equation is utilized for the enlargement or reduction of the altar's size. The Vedic Hindus knew that the numbers $\sqrt{2}$ and $\sqrt{3}$ are irrational. Although there is no explanation in the Vedas how that was discovered, several derivations of their approximate values are embedded in the text itself. Other favourite mathematical topics in the Vedas are permutations and combinations. A special method for finding the number of combinations, called meru prastara, is described in Chandah sutras (200 BC). It is basically the same triangular array commonly known as Pascal's triangle.

The most important Hindu achievement is the decimal positional system. Let me point out that in European mathematics, the decimal system appears only after the 14th century, and the notions of subtraction and zero were not introduced until the 16th century. All of the quantities in European mathematics had dimensions and purely geometric characters.

It is amazing how advanced and sophisticated Hindu mathematics was, a thousand years before the development of European mathematics. In the Hindu decimal system, there are nine symbols called anka (which means 'mark') for the numerals from one to nine, and the zero symbol called sunya (which means 'empty'). The Hindu name for addition was Samkalita, but the terms Samkalana, Misrana, Sammelana, Praksepana, Ekikarana, Yukti etc., were also used by some writers. Subtraction was called Vyukatkalita, Vyutkalana, Sodhana, and

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Patana; multiplication was called Gunana, Hanana, Vedha, Ksaya; and division, regarded as the inverse of multiplication, was called Bhagahara, Bhajana, Harana. The remainder was called Sesa or Antara, and the quotient Labdhi or Labhdha. While European mathematics even in the 16th century did not consider powers of a degree higher than three (since they were not making sense from the geometric point of view), centuries earlier Hindu mathematics studied algebraic equations of degree six or higher. There was even a symbol used for the unknown, which was called Varna, and the unknown quantity was called Yavat-Tavat. Equations with one unknown were called Eka-Varna-samikarana, and equations with several unknowns were called anekavarna-samikarana. There are many more examples of advanced mathematical knowledge contained in the Vedas.

The Vedic methods in arithmetic that were discovered by Sri Bharati Krsna Tirthaji are astonishing in their simplicity. For example, multiplication of large numbers can be done in such an easy way that all the computations and the answer can usually be written in just one line. People who grasp some of the Vedic techniques sometimes dazzle audiences, pretending to be prodigies with a supernatural ability to do complicated computations quickly in their minds. However, it is important to note that no special talent is needed and anybody can take advantage of these ancient methods to improve his or her arithmetic skills. Let me emphasize that many of the mathematical methods described in the Vedas were previously unknown and created great amazement among scholars. In comparison, the circumstances surrounding the discoveries of many ancient Greek or Roman manuscripts dealing with mathematics are considered to be rather suspicious. None of these ancient manuscripts contained any "new" mathematical knowledge, previously unknown to the scientists. This is not the case for the Vedas, which continue to be analyzed, leading to new revelations.

The Vedic methods are direct, and truly extraordinary in their efficiency and simplicity. They reflect a long mathematical tradition, which produced many simplifications, shortcuts and smart tricks. Arithmetic computations cannot be obtained faster by any other known method.

Example 1. A simple idea for factorization of polynomial expressions of two or more variables is rooted in Adyamadyena Sutra—*Alternate Elimination and Retention.* Let us consider, for example, the polynomial $P(x,y,z) = 2x^2 + 6y^2 + 3z^2 + 7xy + 11yz + 7xz$, which can be factorized by setting $z = 0$:

$$P(x,y,0) = 2x^2 + 7xy + 6y^2 = (2x + 3y)(x + 2y), \qquad (1)$$

and next, setting $y = 0$:

$$P(x,0,z) = 2x^2 + 7xz + 3z^2 = (2x + z)(x + 3z). \qquad (2)$$

By comparing the obtained factorizations (1) and (2) and completing each factor with the additional terms from the other factorization, we obtain the factorization of $P(x,y,z)$:

$$P(x,y,z) = (2x + 3y + z)(x + 2y + 3z). \qquad (3)$$

Also, notice that on substituting $x = 0$, we obtain $P(0,y,z) = 6y^2 + 11yz + 3z^2 = (3y + z)(2y + 3z)$, in accordance with the factorization (3).

Example 2. It is also possible to eliminate two variables at a time. For example, consider the polynomial $Q(x,y,z) = 3x^2 + 7xy + 2y^2 + 11xz + 7yz + 6z^2 + 14x + 8y + 14z + 8$. Such eliminations lead to

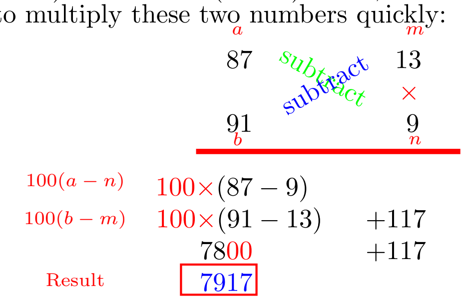
$$Q(x,0,0) = 3x^2 + 14x + 8 = (x + 4)(3x + 2)$$
$$Q(0,y,0) = 2y^2 + 8y + 8 = (2y + 4)(y + 2)$$
$$Q(0,0,z) = 6z^2 + 14z + 8 = (3z + 4)(2z + 2).$$

Using a completion method similar to Example 1, we obtain

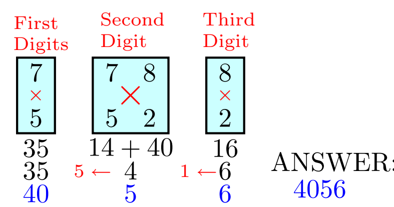
$$Q(x,y,z) = (x + 2y + 3z + 4)(3x + y + 2z + 2).$$

It is easy to verify that this is indeed a factorization of the polynomial $Q(x,y,z)$.

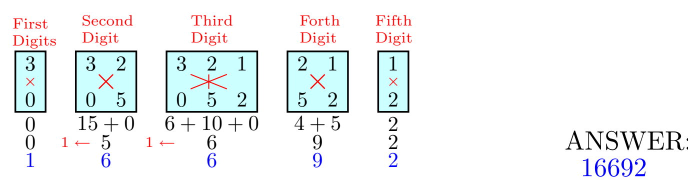
Example 3. In conventional arithmetic, there is no shortcut to multiplying the number $a = 87$ by $b = 91$; this can be done only by 'long multiplication.' But the Vedic method sees these numbers are close to 100 (i.e., the numbers $m = 100 - a$ and $n = 100 - b$ are relatively small). Since $a \cdot b = a(100 - n) = 100(a - n) + mn = 100(b - m) + mn$, there is a very simple way to multiply these two numbers quickly:



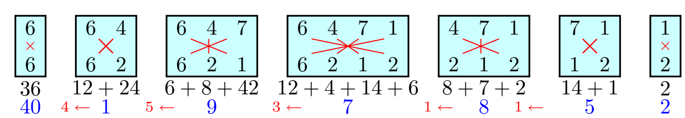
Example 4. Another way of doing a similar multiplication is illustrated below, where we show how to compute the product 78×52 using vertical and crosswise multiplication:



This method can be used to multiply large numbers as well. Let us, for instance, compute the product 321×52:



The product 6471×6212 can be computed in a similar way:



We obtain the answer 40197852.

There are several books written about this fascinating subject, including Vedic Mathematics, by Jagadguru Swami Sri Bharati Krsna Tirthaji Maharaja. Also see the Internet web sites `http://www.vedicmaths.org` and `http://www.mlbd.com`. In many schools, the Vedic system is now being taught to students. "The Cosmic Calculator," a course based on Vedic math, is part of the National Curriculum for England and Wales.